Periods and the multiple gamma function in the $p$-adic case

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$$\frac{d}{ds}\zeta_r(s,u,z)\Big|_{s=0} = \log\left(\frac{\Gamma_r(z,u)}{\rho_r(u)}\right) \sim X(c) \sim g_{K/F}.$$

(for $P_K$) Shimura period symbol.

Conj: can express all CM-periods by $g_{K/F}$.

1. p-adic multiple $\Gamma$-function

$F$: # fld. fix $F \hookrightarrow \bar{\mathbb{Q}} \hookrightarrow \mathbb{C}$, $\bar{\mathbb{Q}} \hookrightarrow \mathbb{C}_p$.

- p-adic topology on $F$
- $|p|_p = \frac{1}{p}$.
- $F_{>0} := F \cap \mathbb{R}_{>0}$

For $z \in F_{>0}$, $\underline{u} = (u_1, \dots, u_r) \in F_{>0}^r$

s.t. $|z|_p > |u_i|_p$ $\forall i$.

$$\zeta_{(p),r}(s,u,z) = \sum_{\underline{n}=(n_1,\dots,n_r)\in\mathbb{Z}_{\geq 0}^r} \left((z+\underline{n}\cdot{}^t\underline{u})\,|z+\underline{n}\cdot{}^t\underline{u}|_p\right)^{-s}$$

Cassou-Noguès:

$\zeta_{(p),r}(s,u,z)$ p-adic analytic at $s=0$.

$\zeta_{(p),r}(k,u,z) = \zeta_{(p),r}(-k,u,z) \in \bar{\mathbb{Q}}$ for $k \in \mathbb{Z}_{\geq 0}$, $k \equiv 0 \bmod N \in \mathbb{Z}_{>0}$.

Def: $L\Gamma_{(p),r}(z,u) = \zeta'_{(p),r}(0,u,z)$.

$$\zeta(s) \quad \prod_p \left(1-\frac{1}{p^s}\right)^{-1} = \sum_n \frac{1}{n^s}$$

$$\zeta'(s) = -\log n \cdot \sum_n \frac{1}{n^s}.$$

=.

gamma $\quad \Gamma(s) = \int_0^\infty e^{-t}\, t^s \frac{dt}{t}$ $\quad (\mathbb{R}_{>0}^\times)$

$$\zeta(s)\Gamma(s)\cdot\pi = \int \sum_{n\in\mathbb{Z}} e^{nt^2}\cdot t^{-s}\frac{dt}{t}$$

$$\theta(z) = \sum_{n\in\mathbb{Z}} e^{nz^2} \qquad \int_0^\infty \theta(t)\, t^s \frac{dt}{t} =$$

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4. Gross Conj:

$\chi \in \hat{G}_T -$

$L_p(s, \chi\omega)$ p-adic L-fcn (Deligne-Ribet, P. Cassou-Noguès)

$\exists\ \zeta_{p,F}(s,c)$ $\quad L_p(s,\chi\omega) = \sum_{c \in \underbrace{C_{f(\chi),(p)}}_{\text{some cone}}} \chi\omega(c)\zeta_{p,F}(s,c)$

Put $\underset{\mathfrak{P}_F}{\overset{\|}{\{\mathfrak{P}_1, \ldots, \mathfrak{P}_t\}}}$ the set of all primes of F, lying above (p).

Gross Conj: Assume (*)

$$\forall \chi \in \hat{G}_T -, \quad \frac{L_p'(0,\chi\omega)}{L(0,\chi)} = \frac{1}{2h_K} \prod_{i=2}^{t} (1 - \chi(\mathfrak{P}_i)) \sum_{\tau \in G} \chi(\tau) \log_p N_{K_{\mathfrak{P}_i}/\mathbb{Q}_p} \left( \alpha^{\tau\rho} / \alpha^{\tau} \right).$$

$=$.

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§5. p-adic periods.

$P_K$ defined by decomposing $\int_c \omega_r$ (c.f. Thm 5.1 in Yoshida's lecture.)

can be interpreted in terms of CM motives / K associated to Hecke characters (Blasius)

& $I_\infty : H_B(M) \otimes \mathbb{C} \cong H_{dR}(M, \Omega^1_{\mathbb{R}}) \otimes \mathbb{C}$.

Use p-adic comparison isomorphisms: ($I_\infty$ by $I_p$) ...